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ADVANTAGES AND DISADVANTAGES OF THREE MATHEMATICAL MODELS OF A PEAR BORDER

Snezana MATIC-KEKIC*, Nebojsa DEDOVIC

[1] University of Novi Sad, Faculty of Agriculture, Department of Agricultural Engineering, Lecture desk of Mathematics, Trg D. Obradovića 8, 21000, Novi Sad Serbia
*(Corresponding author: snmk@polj.uns.ac.rs)

Abstract

Various factors have influence on the growth and development of bio-materials. Consequently, shape variability is very important and should be examined. In many processes of heat exchange, as well as in other processes in bio-material handling, the physical properties of a fruit such as dimensions, shape, surface area and volume play significant role. The purpose of this study is to find a function which approximates a pear border line as precisely as possible. One type of estimation of an average pear border line was relying on the sixth order polynomial and proposed algorithm. Also, another two different ways of calculating the Williams pear border line were shown earlier. The first one included spline functions for an estimation of a pear border line, while the second way used regression function obtained by the nonlinear regression method. The regression function had two independent variables, the length and total length of a pear. Border lines of all pears in the sample were fitted with one regression function with large precision (R^2=97.48). The surface area and volume of a pear were calculated based on the regression function and total pear length. In this paper, it is compared three different ways of pear border line calculation.

Key words: shape variability, integral calculus, cubic spline, nonlinear regression

Introduction

Physical properties of fruit (dimensions, shape, surface area and volume) have very important role in many processes of heat exchange and other processes of biomaterial handling (Mohsenin, 1980). It is well known that fruits, including pears, are dominantly irregular in shape. Certain number of measurements must be made for full characterization of fruit shape. Tne analysis of three mutually perpendicular axes usually contains enough information for the volume or surface area modeling. The finite element method was used to discretize the governing differential equations over the actual 3D pear geometry (Wang *et al*., 2006). The pear dimensions were evaluated during the drying process and those data were used to calculate the pear surface area and volume. Cut pears were photographed horizontally and vertically against a millimeter-scaled paper. The shape of the whole pear was replaced with two regular bodies, half of a sphere and a cone (Guine *et al*., 2006). The dimensions and volume were also investigated for cherries (Ochoa *et al.*, 2007), almond cultivars (Altuntas *et al*., 2010) and mango (Spreer and Müller, 2010). If color is an important factor, then the use of digital images is essential (Quevedo *et al*., 2009; Purlis and Salvadori, 2009; Altuntas *et al.*, 2010;). Xiao-bu *et al.* (2010) also used digital images to detect the apple defects. Optimization of digital images was widely studied (Acketa and Matić-Kekić, 2000; Matić-Kekić *et al.*, 1996). An image processing-based method is appropriate for measuring the volume and surface area of ellipsoidal agricultural products such as lemons, peaches, limes and eggs (Sabliov *et al*., 2002).

Now, let us make a short introduction to the one of three methods, that are going to be presented here. For various kinds of approximation problems, it is frequently advantageous to use piecewise polynomials instead of polynomials because using low-degree polynomials locally is usually more accurate and more efficient than using a high-degree polynomial globally (Ascher *et al*., 1995). The main property of cubic spline function is that it remains twice continuously differentiable over the observed interval. Program package Mathematica 6 (Wolfram, 1991) was employed for testing the cubic spline approximation of pear border line and for all necessary numerical integrations. This software is also very applicable in many other problems related to agriculture (Bodroža-Pantić *et al.*, 2008) and to optimization (Matić-Kekić and Acketa, 1997; Acketa *et al.*, 2000).

The purpose of this study was to compare various mathematical expressions for Williams pear border line. Those expressions allow easy estimation of the pear surface area and volume. Dedović *et al.* (2011) confirmed the precision of mathematical model for the pear border line approximation using the following procedure: 1, volumes of the pears were measured by Archimedes' method; 2. volumes were calculated using numerical integral calculus with assumption that pear could be observed as a rotation body; 3. relative errors for calculated volumes were given.

Material and Methods

Thirty fruits of Williams pear (*Pyrus communis*) were randomly selected and then halved through the longitudinal axis. Each half was split along the same axis to generate two pear quarters. The core and seeds were removed and the half of a pear was placed in a two-axis system, such that the total length (L) of the pear was on l-axis, and the width (W) was on the w-axis. Zero point was placed at the bulbous end of the pear (see Fig. 1).

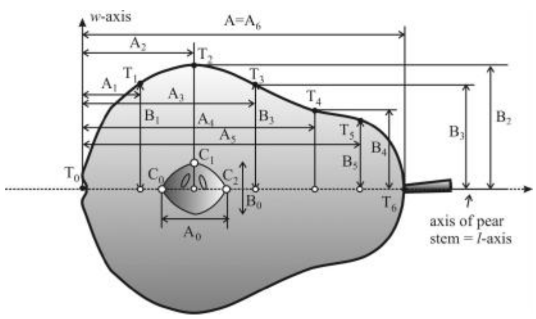

Figure 1.[1] The pear outline and its measured points T_i (A_i,B_i), i=1,…,6, as well as dimensions of pear core A_0 and B_0. Point $T_0(0,0)$ is start point of Cartesian coordinate system with l-axis (length) and w-axis (width); $A_1=0.5\cdot A_2$, $A_3=0.5\cdot(A_2+A_4)$, $A_5=0.5\cdot(A_4+A_6)$, $C_0=(0.5\cdot(A_2-A_0),0)$, $C_1=(A_2,B_0/2)$ and $C_2=(0.5\cdot(A_2+A_0),0)$.

The coordinates of the seven points, located on the pear border line $T_i(A_i,B_i)$, i=0,…,6, are presented in Babić *et al*. (2012) and Dedović *et al*. (2011).

Among basic dimensions, thickness of the pears was not measured because it was assumed (www.rainierfruit.com/products/pears/img/pears.pdf) that pear could be represented as rotating body, where "stem axis" was, actually, the axis of rotation. According to this assumption, pear thickness is equal to pear width, and the surface area and volume of the pear

[1] This figure is taken from Babić *et al*. (2012).

can be calculated using formulas (1) and (2), respectively. However, the pear border line function $f(x)$ must be defined first. Generally speaking, the formulas for the surface area S and volume V of the rotating body within the interval (a,b), where a is the starting point on the x-axis and b the last point on the same axis, for a non negative period of function $f(x)$ are:

$$S\left(f(x),a,b\right)=2\pi\int_a^b f(x)\cdot\sqrt{1+\left(f'(x)\right)^2}\,dx \qquad \text{(the surface area of rotating body)} \qquad (1)$$

and

$$V\left(f(x),a,b\right)=\pi\int_a^b\left(f(x)\right)^2 dx \qquad \text{(the volume of rotating body)} \qquad (2)$$

The volumes of pears from the sample were measured by Archimedes’ method and calculated by formula (2), where $f(x)$ was appropriate border line function (polynomial function (4), spline function (5) or regression function (6)). Also, the surface area of pears from the sample, were calculated by formula (1), where $f(x)$ was border line function.

The quarter of one pear was additionally bounded with two lateral flat surfaces. Flat surface area, denoted as FS, bounded by the x-axis and $f(x)$, was calculated as:

$$FS(f(x),a,b)=\int_a^b f(x)\,dx \qquad (3)$$

for $f(x) \geq 0$, $x\in(a,b)$. The surface area and volume of a seed core, can also be calculated by using (1) and (2) since seed core can be considered as a rotating body, too.

Results and Discussion

1. Polynomial fitting

In Babić *et al*. (2012), $f(x)$ is the function which approximates the pear border line passing through the seven points T_i, i=0,1,2,…,6 on the average pear border line. The function $f(x)$ is, actually, sixth order polynomial $P(l)$,

$$P(l) = 4.1135\cdot l - 0.2531\cdot l^2 + 0.0093\cdot l^3 - 0.0002\cdot l^4 + 2.083\cdot 10^{-6}\cdot l^5 - 8.5968\cdot 10^{-9}\cdot l^6 \qquad (4)$$

where $l\in[0,\overline{L}]$ is an independent variable, i.e. l is pear length which takes values from 0 to $\overline{L}$, while the average total length of pears is marked as $\overline{L}$=84.3 mm.

2. Spline functions

The next objective is to determine the polynomial function of lower order than sixth, with the same outline representation. High order of polynomial $P(l)$ produces very small coefficients which multiply l^5 and l^6, causing the possible less precision and greater errors during the numerical calculations of the volumes and surfaces area. If cubic spline involves all seven characteristic points T_i, i=0,1,…,6, the interval [0,84.3] has to be split into four parts. The first subinterval [0,29] contains points T_0, T_1 and T_2, the second one [29,44.5] contains T_2 and T_3 points, the third subinterval [44.5,60] contains points T_3 and T_4, and the fourth subinterval [60,84.3] contains points T_4, T_5 and T_6. Thus, cubic spline $s(l)$ in (5) is represented by four 3rd order polynomial functions (one polynomial for each subinterval).

In the case when total length of a pear is L=84.3, the function describing pear border line is

$$s(l)=\begin{cases} 3.09764\cdot l-0.0984982\cdot l^2+0.00102846\cdot l^3,\ l\in[0,29], \\ 30.1456-0.011803\cdot l+0.00841199\cdot l^2-0.000196814\cdot l^3,\ l\in[29,44.5] \\ -75.7114+7.10387\cdot l-0.151026\cdot l^2+0.000994001\cdot l^3,\ l\in[44.5,60] \\ 628.3-27.9943\cdot l+0.432242\cdot l^2-0.00223695\cdot l^3,\ l\in[60,84.3] \end{cases} \qquad ((5)$$

Two previously described methods were based on the same idea. Firstly, the functions which approximate average pear border line (polynomial or spline functions) pass through all seven points on the average pear outline. Secondly, those functions are later used for stretching or compressing of pear border line for each pear, using the previously proposed algorithm. However, there is the following limitation: each total pear length requires new polynomial or spline function to be created (meaning that each pear from the sample has a shape which can be obtained by stretching or compressing the average pear border line). In general, it cannot be correct because the shape of a pear from the same cultivar does not have to be equal to the shape of an average pear. These problems can be overcome by using the following approach.

3. Nonlinear regression

Dedović *et al*. (2011) created only one function (6)

$$Q(l,L)=-\frac{40.6437}{L^5}\cdot l^6+\frac{116.805}{L^4}\cdot l^5-\frac{130.088}{L^3}\cdot l^4+\frac{72.6492}{L^2}\cdot l^3-\frac{22.9904}{L}\cdot l^2+4.26753\cdot l \qquad ((6)$$

which described the variability of pear boarder line. Very high coefficient of determination R^2=97.48% was obtained for the confidence level of 95%. The main advantage of this model is that there is no need for extra algorithm given in Babić *et al.* (2012). Another advantage is that this (third) method is more precise than the first two methods previously described because all experimental points were used in nonlinear regression method and not only the mean values of seven coordinates on the pear border line.

Conclusions

The Williams pear border line is approximated by: polynomial function, spline function and nonlinear regression function.

Cubic spline has similar properties as the polynomial function for the average pear border line approximation. The only advantage of cubic spline is more acceptable polynomial coefficients.

Pear border line variability implies that priority should be given to the nonlinear regression function for the following reasons:

it calculates pear border line for smaller and larger pears more precisely than other two proposed functions;

the volumes obtained by using the regression function as a pear border line approximation differs from the exact volumes (measured by Archimedes’ method) with relative error 6.97%. Removing the three smallest pears from the sample, relative error is then 4.24% only (Dedović *et al*. 2011). Very high precision implies that surface area of a pear can be calculated very precisely as well.

it is clear that precise determination of the pear border line approximation, implies that calculation of the surface area and volume of a pear will be more accurate;

only one function is sufficient for pear border line formation, for arbitrary total pear length;

regression function does not require any additional algorithm.

These results are useful for packing, storage as well as in technology processes, biomaterial handling and drying processes.

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